Plamsa Formation Mechanisms in Exploding Wire Experiments
Final Report for Spring 2003

Jeffrey A. Vaughan
Laboratory for Plasma Studies
Cornell University

October 6, 2003

1 The Experiment

The Low Current Pulser generates up to a 30 kV potential difference between the wire holder's anode and cathode. This voltage drops across the specimen and is supported by a combination of the wire's resistance and the holder's inductance. A typical load may have 2Ω resistance and 50 nH inductance.

Voltage and current are measured with each shot. Current is recorded using a rogowski coil and voltage with a resistive divider. Modeling the wire and wireholder as an LR circuit (fig. 1) we measure total voltage V_0. It is straightforward to calculate the resistive component, V_r, using the numerical derivative of current, a measured inductance and the relation

$$V_0 = V_r - L\frac{dI}{dt}$$

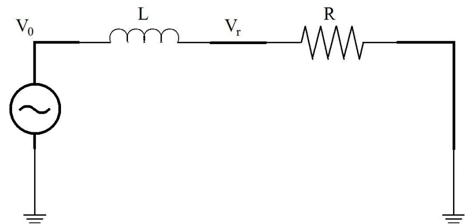
In practice we have found our current traces to be noisy. We do not believe this is real, and use a Fourier filter to strip current frequency components above 50 MHz. Like the rest of the V_r calculation, this is done numerically.

2 Proposed mechanisms

One interpretation of the preliminary data follows.

Our data shows both voltage across and current through the wire holder increases at the beginning of a shot. After some time, the voltage drops sharply, while the current remains high. It is currently hypothesized that the voltage collapse corresponds to a change in conduction mode. That is, before collapse the current flows through the wire, which behaves like an ohmic material; after collapse the current is carried by plasma.

Figure 1: A simple circuit model for the Low Current Pulser shows the wire as resistor R.



Two separate processes may generate plasma. Plasma may form the ends of the wire, due to electrical arcing, or along the wire as consequence of the high transverse **E** field. We propose that there is a "race condition" whereas both effects may occur, but only a single one is expressed in timing data. In this report, I will refer to these as end and transverse effects.

2.1 End Effects

If the anode-wire or cathode-wire connections are of poor quality, current will have a tendency to arc at the connection point. This will create a localized neutral plasma consisting of both positive metal ions and free, highly mobile, electrons. The large potential difference between the wire holder electrodes implies a large **E** field which accelerates electrons toward the anode. While actually mobile, the ions' large mass retards their acceleration, and I consider their motion negligible compared to that of the electrons.

A simple kinematic argument demonstrates that end-effect plasma can cross the wire holder quickly. The actual electron flow dynamics are complicated, and should be investigated by computer simulation. However we can reason about the condition at voltage collapse. Before collapse there is no plasma between the wire holder plates. If we treat applied voltage as a slowly varying quantity we can model the gap as a region with constant electric field. That is:

$$|\mathbf{E}| = \frac{V}{d}$$

where d is the anode to cathode separation and V is the applied voltage. Ignoring relativity, applying Newton's second law to the Lorentz force, and integrating acceleration we find

$$t^2 = \frac{2d^2 m_e}{Ve^-}.$$

Here t is time for a single electron to cross the gap, and m_e and e^- represent the electron's mass and charge. If we assume that space charge effects are small for the first electrons crossing the gap, this formula lets us predict the minmal possible time between plasma formation and voltage collapse. For a 1 cm gap I have found $t \geq 0.34$ ns and, for a 3 cm gap, $t \geq 1.01$ ns.

The simplified end-effect plasma transport implies that the time required for electrons to cross the wire holder mat be very small regardless of wire length. A consequence of this is that edge-effect based voltage breakdown should occur almost instantly after an initial arc at the wire connection site. Therefore, we would expect to see breakdown (when caused by end effect plamsa) at a single voltage regardless of wire length.

2.2 Transverse Effects

Transverse effect plasma is generated along the entire length of the wire during a shot. This plasma need not be homogeneous in space, and laser back-lit photographs indicate that is is not.

The physics behind transverse effect plasma formation appears deciptively straightforward. In the beginning of a shot, the wire acts as an ohmic resistor and undergoes Joule (I^2R) heating. As this process continues, parts of the wire will melt, boil and finally ionize. Electrons liberated by ionization escape and form a plasma shell ground the wire's remnants.

This process will be governed by energy deposition; a unit length of wire will generate plasma only after absorbing some critical energy E_0. A naive approach to finding E_0 is to assume thermodynamic equilibrium and claim that for a wire with length l and linear density λ:

$$E_0 = l\lambda \int_{T_0}^{T_i} H(T')dT'$$

The integral's bounds indicate that we are starting at the wire's initial temperature and increasing T' until we reach the the lowest temperature at which the wire becomes a plasma. Here $H(T')$ represents the the heat required to to raise the temperature of a unit mass by one degree. This generalized function is intended to fold heat capacity and phase change energies (e.g. latent heat of fusion) into a simple form and, in MKS, has units Joules / Kelvin. An appropriate $H(T')$ can be assembled from published material properties.

Unfortunately, the actual value of E_0 is difficult to calculate. This is because the above discussion assumed thermodynamic equilibrium. However wire, explosions happen quickly and some parts of the wire may undergo phase changes before others. This seems especially likely in view of the fact that current flows on the outside of conductors. It is easy to imagine the outside layer of the wire ablating while the center remains relatively cool.

Fully incorporating this analysis into the expression for E_0 would add significant complexity. Instead, I add a zeroth order scaling factor, ξ, and find

$$E_0 = \xi l\lambda \int_{T_0}^{T_i} H(T')dT'$$

Assuming a set of experiments is run with similar (same composition and gauge) wires, both λ and the integral are clearly constant. Therefore

$$E_0 \propto l \tag{1}$$

Although, the final gas to plasma transition will almost certainly be triggered by an Paschen effect avalanche, and will not be a simple function of energy deposition, I will neglect this distinction.

Continuing this analysis, we will consider the case of a linear current pulse applied to a wire with fixed length. We can write current as function of time

$$I(t) = \frac{dI}{dt}t = kt$$

where k is the slope of the current. The quantity $I(0)$ vanishes because we define $t = 0$ as the beginning of the current pulse. The wire, with resistance, r, will undergo Joule heating at rate

$$\left.\frac{dE}{dt}\right|_t = rI^2(t)$$

and, by breakdown at time t_b, absorb total energy

$$E_0 = \int_0^{t_b} rI^2(t)dt = k^2 \int_0^{t_b} rt^2 dt \approx rk^2 \int_0^{t_b} t^2 dt$$

The last step comes from treating resistance as

$$r(t) = r + (\text{higher order terms})$$

and dropping the non-constant terms. The validity of this approximation depends on the wire's resisitvity as function of temperature, which is a measured quantity. Unfortunately experimental evidence suggests that the approximation may not be sound. Integrating, rearranging, and substituting for k yields

$$\frac{3E_0}{r} = k^2 t_b^3$$

$$t_b \propto \left(\frac{dI}{dt}\right)^{-2/3} \tag{2}$$

2.3 Summary

Although approximate, the preceding analysis suggests several proprieties of a z-pinch system governed by the proposed mechanism. We found that end effect plamsa generation should be primarily governed by voltage level, and that its formation and transport are not heavily influenced by wire length, energy deposition or pulse shape. In contrast transverse effect plasma formation will be effected by these factors. In particular, energy deposition before breakdown should scale with wire length and breakdown time should scale as described in equation 2. Electric potential aside, nothing in this model distinguishes the anode and cathode. Therofore its predictions are symetric with respect to polarity; the choice of anode or cathode is expected to be arbitrary.

3 Data Analysis

3.1 Timing Data

At the June 2003 LPS workshop, Peter Duselis presented preliminary data including current rise rates (dI/dt) and breakdown times measured during several identical experiments. In figure 2 this data is shown to follow the $-2/3$ power law derived in section 2.2. However this agreement is not perfect, and table 1 gives emperical power laws for various configurations.

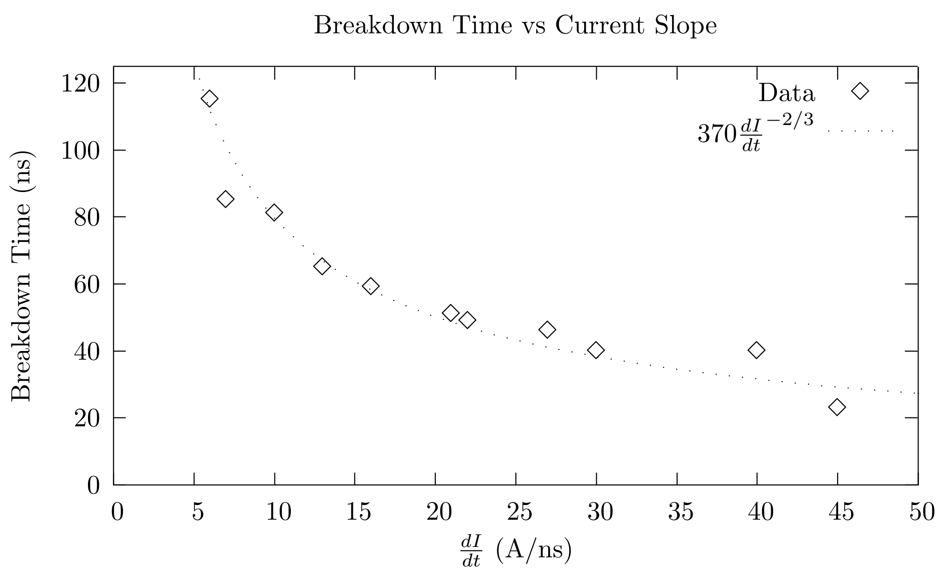


Figure 2: Wire data demonstrates behavior simular to that predicted for transverse effect plasma

Table 1: This table gives power laws relating breakdown time to current rise rate. The relation is $t_b \propto (dI/dt)^p$.

Drive Current	Ends	Power (p)
Positive	Unsoldered	-0.53
Positive	Soldered	-0.54
Negative	Soldered	-0.61
Negative	Unsoldered	-0.66

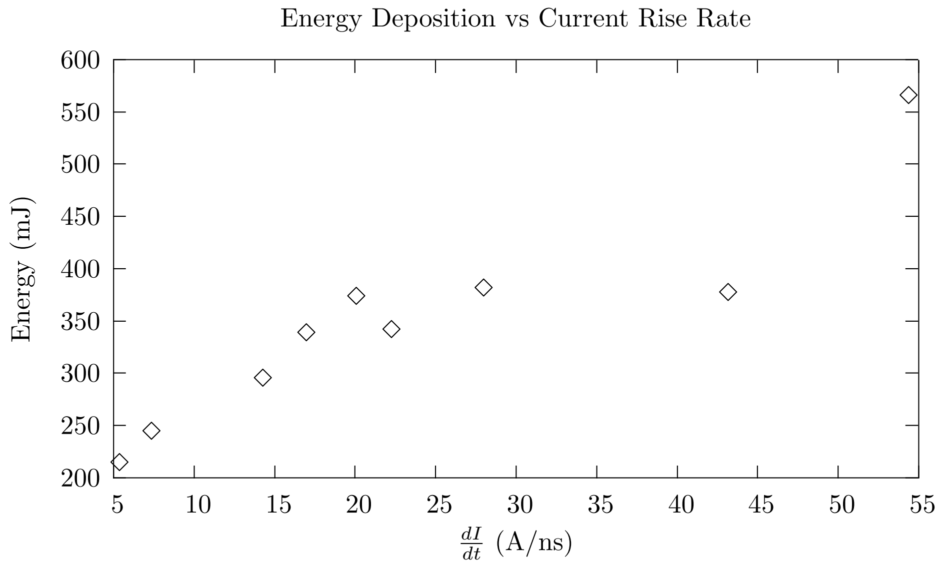


Figure 3: Energy deposition as a function of $\frac{dI}{dt}$ for shot 3 cm, 25 μm copper wires.

3.2 Energy Deposition

The model assumes that the system evolves under near equilibrium conditions and uses a simple thermodynamic argument to calculate energy required to generate plasma. As a result of these assumptions, equation 1 implies energy deposition is a fucntion only of wire length. The data yields a different result. Figure 3 shows that energy deposited before breakdown varies directly with current rise rate.

3.3 Breakdown Voltage

While transverse effect plasma breakdown voltage is clearly dependant on current rise rate, the model predicts that end effect plasma breakdown voltage is a function of geometry alone. Figure 4 shows breakdown voltage monotonically inceasing given a single experimental geometry and a range $\frac{dI}{dt}$ values. This plot suggests voltage collapse may not be governed by end effect plasma. While this does not support the "race condition" model, it is not a conclusive refutation.

3.4 Polarity Dependence

In contradiction to the proposed model, drive current polarity significantly effects the exploding wire system. For unsoldered wires, table 1 shows that changing from negative to postive drive polarity changes the breakdown time power

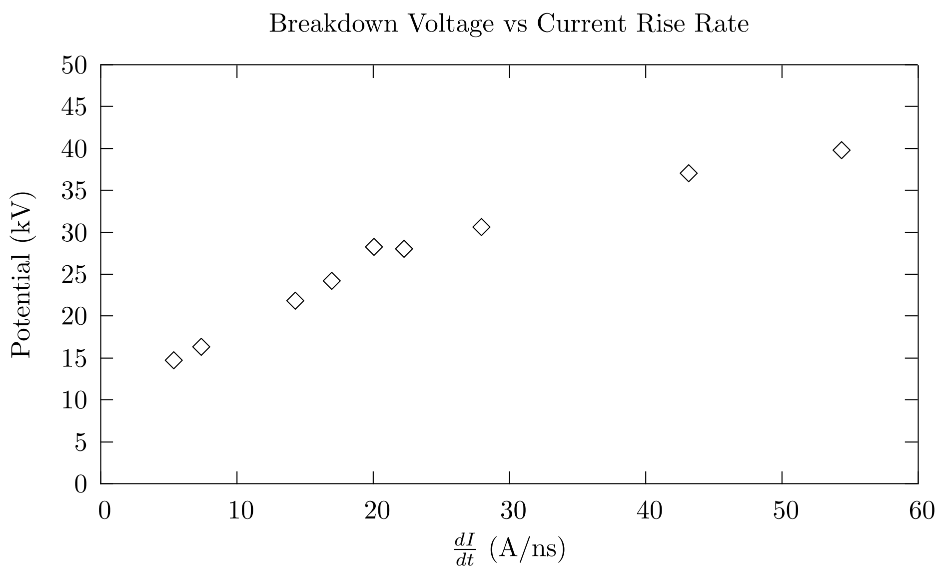


Figure 4: Breakdown voltage as a function of $\frac{dI}{dt}$ for shot 3 cm, 25 μm copper wires.

law (eq. 2) by a factor of $(dI/dt)^{0.13}$. This is a change of 24.5% in the exponent and does not appear insignificant.

4 Conclusion

The proposed model for exploding wire dynamics attempts to explain voltage breakdown in terms of race condition between end and transver effect plasmas. While it agrees reasonably well with the observed relation between breakdown time an current rise rate, the model makes several predictions which are contrary to the data. In particular, the data shows that breakdown energy is dependant on current rise rate and that the wire sustem is not symetric in drive polarity.

For these reasons, the proposed model is probably too simple. A complete description will need to include wire holder geometry, dynamic wire resistivity and more complete plasma physics.

5 Acknowledgements

I would like to thank Professor Bruce Kusse and Peter Duselis for insightful conversation and pratical laboratory instruction. In addition, I am grateful for Professor Kusse's unending patience regarding this report.