A Cryptographic Decentralized Label Model

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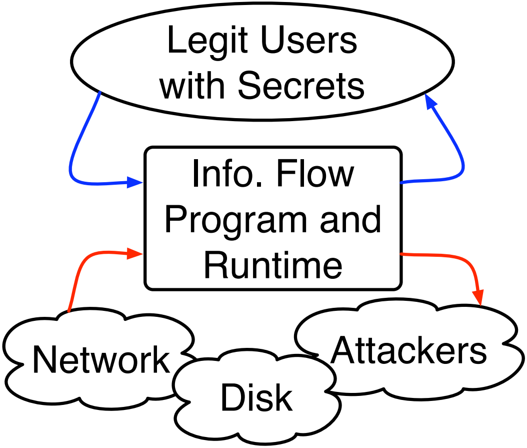
Information flow protects secrets from disclosure.



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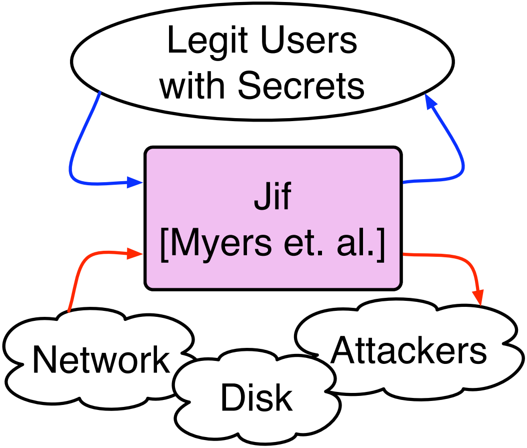

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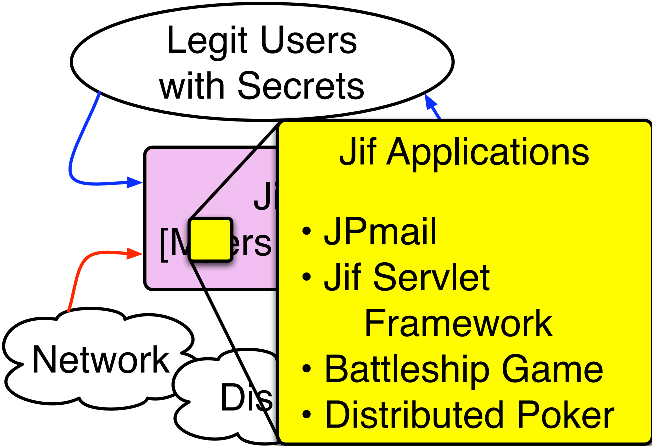

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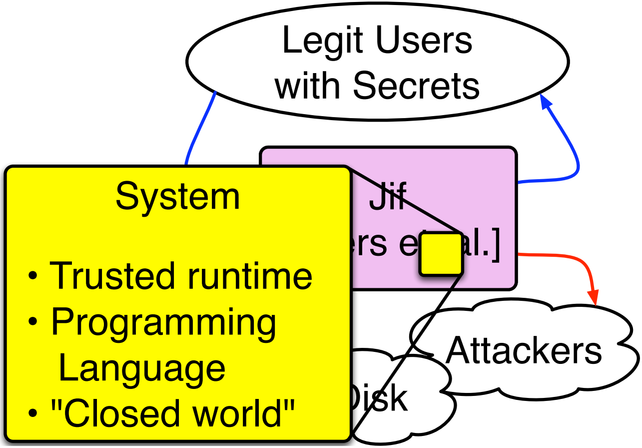

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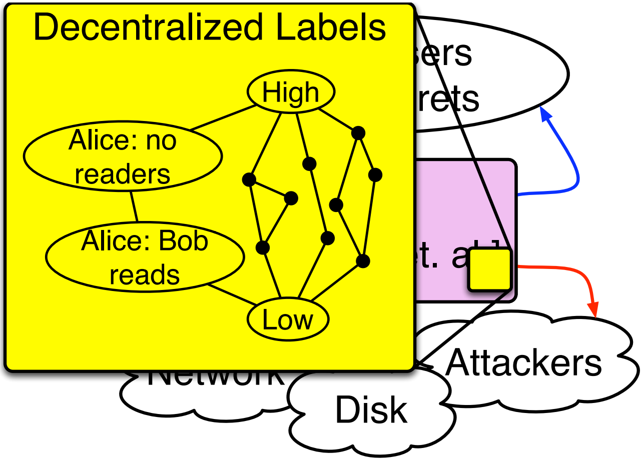

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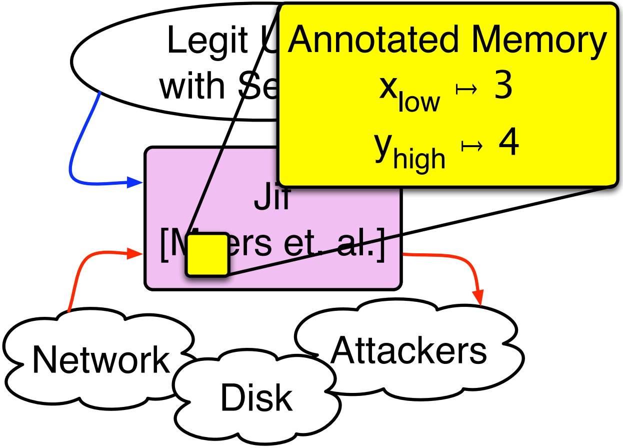
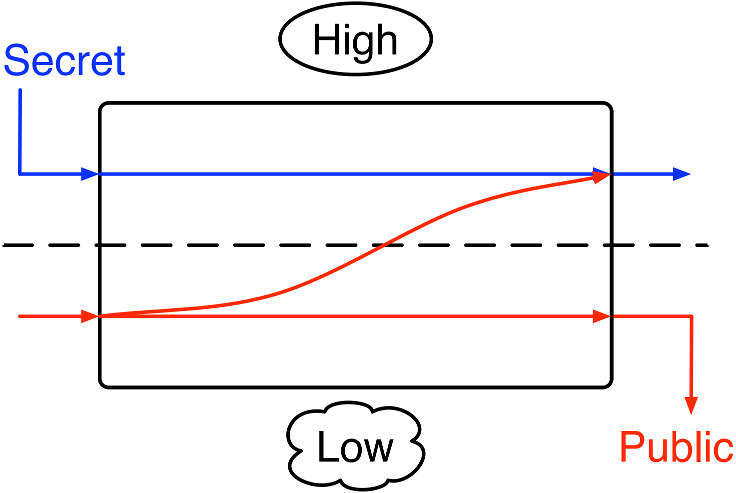


Noninterference: high inputs don't affect low outputs



[Denning & Denning CACM '77] [Pottier & Simonet TOPLAS '03]
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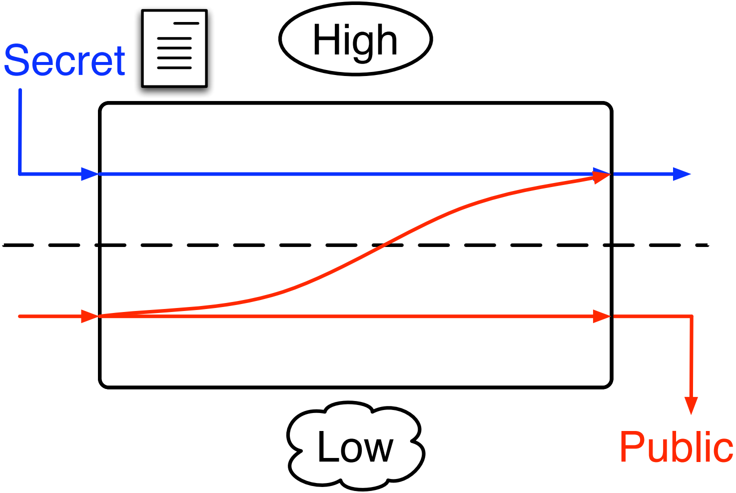

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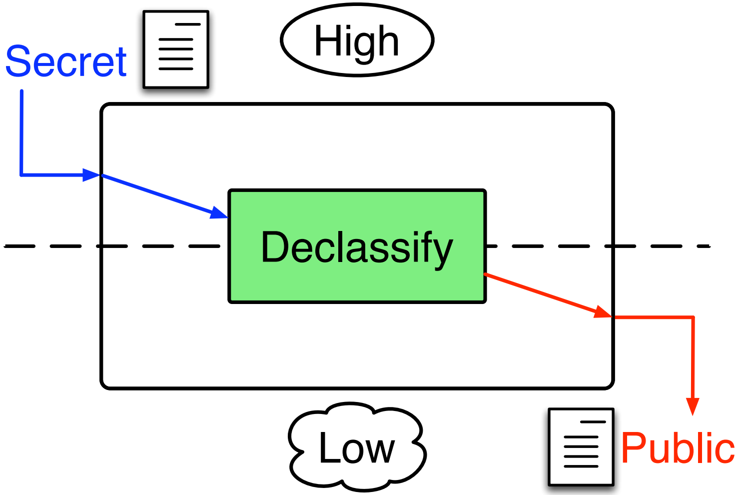
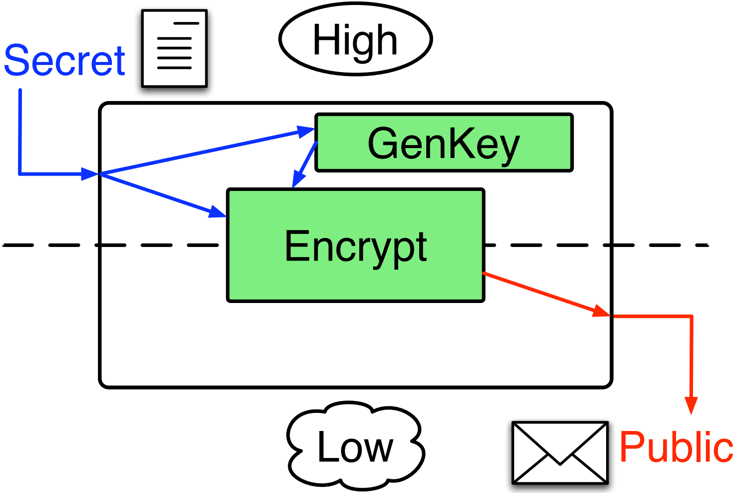
Some programs need to violate noninterference.



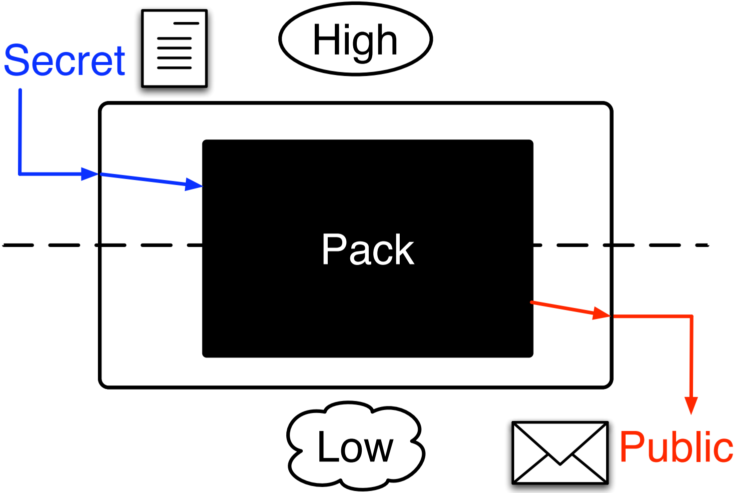
Satisfies (decentralized) robust declassification instead of noninterference [Myers & Chong CSFW '06].

Encryption can restore noninterference.



[Chothia, Duggan, & Vitek CSFW ’02] [Sumii & Pierce POPL ’04]
[Laud & Vene ACSAC ’05] [Askarov, Hedin, & Sabelfeld ISAS ’06]

Our idea: make the cryptography transparent.



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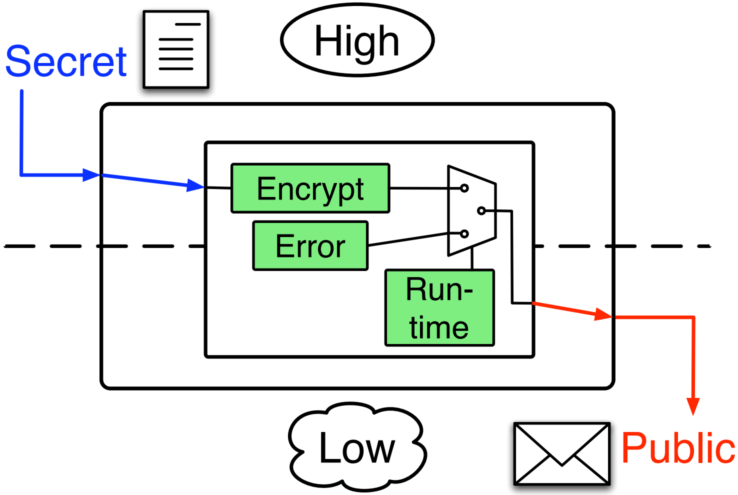

Our solution: label directed implicit packing.

The Cryptographic Decentralized Label Model unifies

- a high-level, information-flow *language*,
- declarative *labels* that describe security policies,
- and cryptographic *packages* that implement policies.

Key notation

$$\begin{array}{ccccc} \text{data} & + & \text{policies} & \Rightarrow & \text{package} \\ v & + & \ell & \Rightarrow & \langle v \rangle_\ell \end{array}$$

SImp language can pack and unpack labeled data.

Definition (SImp Syntax)

types	τ	$::=$	$\texttt{int} \mid \ldots \mid \texttt{pkg}$	
values	v	$::=$	$0 \mid 1 \mid \ldots$	
		$\mid$	$\langle v \rangle_\ell$	package
expressions	e	$::=$	$\ldots$	
		$\mid$	$\texttt{pack}\ e\ \texttt{at}\ \ell$	package intro
		$\mid$	$\texttt{unpack}\ e\ \texttt{as}\ \tau\{\ell\}$	package elim

Packages may be constructed and analyzed according to ℓ.

SImp can implement a simple messaging system.

Example

```
text: string{high}        dest: string{low}
out: pkg{low}             in: pkg{low}

text := readLine()
match (pack text at {high}) with
  ok(p)  => out := p; send(out)
  error  => skip

            ⋮

in := receive()
match (unpack in as text{high})
  ok(t) => text := t;  printLine(text)
  error => skip
```

Pack succeeds iff runtime has sufficient authority.

The SImp runtime contains

- a memory, M, and
- an authority, $\overline{p}$.

Evaluation Model

$$\frac{\text{precondition}}{\text{runtime state} \vdash \text{from} \rightarrow \text{to}}$$

Definition (Pack Evaluation)

$$\frac{\overline{p} \textit{ writes } \ell}{\overline{p}; M \vdash \texttt{pack } v \texttt{ at } \ell \rightarrow \texttt{ok}(\langle v \rangle_\ell)} \quad \textsc{E-Pack-Ok}$$

$$\frac{\neg(\overline{p} \textit{ writes } \ell)}{\overline{p}; M \vdash \texttt{pack } v \texttt{ at } \ell \rightarrow \texttt{error}} \quad \textsc{E-Pack-Fail}$$

Without enough keys, it is *infeasible* for the runtime to perform these operations.

Unpacking also performs dynamic checks.

Definition Fragment (Unpack Evaluation)

$$\frac{\overline{p} \textit{ reads } \ell \qquad \ell_0 \leq \ell \qquad \vdash v_0 : \tau}{\overline{p}; M \vdash \texttt{unpack } \langle v_0 \rangle_{\ell_0} \texttt{ as } \tau\{\ell\} \rightarrow \texttt{ok}(v_0)}$$

Checks ensure

- cryptographic feasibility
- information flow
- type safety (values have correct shapes)

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- type safety (values have correct shapes)

Is failure to pack a covert channel?

Example

```
h: bool{high},    l: bool{low},    v: pkg{?}

if h then
  v := pack 0 at low
else
  v := pack 0 at high
          ⋮
match (unpack v as bool{low}) with
  ok(_) => l := true
  error => l := false
```

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```

Rule: High guard requires variables assigned in branches are high.

Constraints: v is high

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if h then
  v := pack 0 at low
else
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          ⋮
match (unpack v as bool{low}) with ⇐
  ok(_) => l := true
  error => l := false
```

Rule: Unpack does not declassify secret data.

Constraints: v is high; v is low

Is failure to pack a covert channel?

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if h then
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```

Constraints: v is high; v is low; high = low

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```

$\therefore$ reject statically

Pack provides a limited declassify, unpack an endorse.

Definition Fragment (Pack Security Typing, 1st attempt)

If

- *e has label* ℓ_e

then

`pack` e `at` ℓ_e *has label* ℓ.

Definition Fragment (Unpack Security Typing, 1st attempt)

If

- *e has label* ℓ_e*, and*
- *labels* ℓ *and* ℓ_e *are equal,*

then

`unpack` e `as` $\tau\{\ell\}$ *has label* ℓ.

Pack provides a limited declassify, unpack an endorse.

Definition Fragment (Pack Security Typing)

If

- *e has label ℓ_e, and*
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Can we “cast away” security labels?

Example

```
h: bool{high},    l: bool{low},    v: pkg{low}

v := pack h at {high}
          ⋮
match (unpack v as bool{low}) with
  ok(true)  => l := true
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```

Can we "cast away" security labels?

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State: $h = true$

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State: $h = \textit{true}$; $v = \langle \textit{true} \rangle_{high}$

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Check: *high* $\leq$ *low*

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```

∴ unpacking fails

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Conclusion: not a leak

Evaluation respects noninterference.

Property ($M_1 \cong_\ell M_2$)

Memories M_1 and M_2 are equivalent to an observer with power ℓ, if the observer cannot distinguish the memories.

Example

M_1
$x_{low} \mapsto 2$
$y_{low} \mapsto \langle 3 \rangle_{high}$
$z_{high} \mapsto 3$

M_2
$x_{low} \mapsto 2$
$y_{low} \mapsto \langle 4 \rangle_{high}$
$z_{high} \mapsto 4$

$$M_1 \cong_{low} M_2$$

$$M_1 \ncong_{high} M_2$$

Property (Noninterference)

$$M_1 \xrightarrow{\text{evaluation}} M_1'$$

$$M_2 \xrightarrow{\text{evaluation}} M_2'$$

$$M_1 \cong_\ell M_2 \implies M_2' \cong_\ell M_1'$$

SImp is parameterized by a *security lattice*.

Definition Fragment (Security lattice)

A lattice whose elements are composed of orthogonal confidentiality and integrity components is a security lattice.

Example

$$high = \{private!tainted\}$$

$$\{private!trusted\} \qquad \{public!tainted\}$$

$$low = \{public!trusted\}$$

Decentralized labels specify security policies.

$$\ell = \{\mathsf{Alice}: \ \mathsf{Bob} \,!\, \mathsf{Charlie}; \quad \mathsf{Bob}: \ \mathsf{Alice} \,!\, \mathsf{Charlie}, \mathsf{Dave}\}$$

- A label is a list of policies.
- A policy consists of
  - an owner (who may distrust other owners),
  - a reader set, and
  - a writer set.

Property ($\overline{p}$ *reads* ℓ)

Principal set $\overline{p}$ reads ℓ when each owner in ℓ permits some member of $\overline{p}$ to read.

Example

	reads	writes
Alice	✓	x
Dave	x	x
{Alice, Dave}	x	✓

Labeling is a variant of Myers and Liskov's DLM [TOSEM '00].

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Labels and packages need meaning outside of SImp.

- Cryptographic assumptions
  - Each principal is mapped to a well-known public key.
  - Cryptographic functions follow the Dolev-Yao model.
- Goals of interpretation
  - Package confidentiality protects data from eavesdroppers.
  - Package integrity protects the program from data.
  - Packages can created and consumed offline.

Packages compile to cryptographic messages.

Example (Compile $\langle 42 \rangle_{\{\text{Alice: Bob!}\}}$)

1. Generate fresh key pairs (R^+, R^-) and (W^+, W^-).
2. Let $payload = sign(W^-, \{\!|42|\!\}_{R^+})$
3. Let $seal = sign(\text{Alice}, [\texttt{"\{Alice: Bob!\}"}, R^+, W^+, \{\!|R^-|\!\}_{K^+_{\text{Alice}}}, \{\!|R^-|\!\}_{K^+_{\text{Bob}}}, \{\!|W^-|\!\}_{K^+_{\text{Alice}}}])$
4. Return $package = (seal, payload)$

- R^- is a read capability.
- W^- is a write capability.

Package compilation is adequate.

Property ($m_1 \cong_\ell m_2$)

Say $m_1 \cong_\ell m_2$ when messages m_1 and m_2 reveal only equivalent information to Dolev-Yao observers weaker than ℓ.

Lemma (Adequacy for Values)

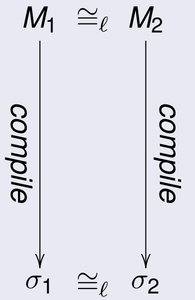
If

$$v_1 \cong_\ell v_2$$

then

$$compile(v_1) \cong_\ell compile(v_2).$$

Corollary



Read the paper for more results.

- Today we discussed noninterference and adequacy.
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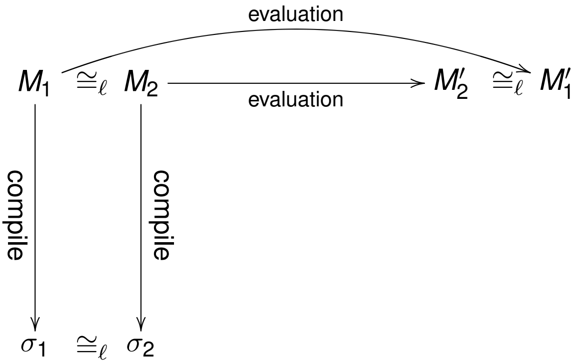

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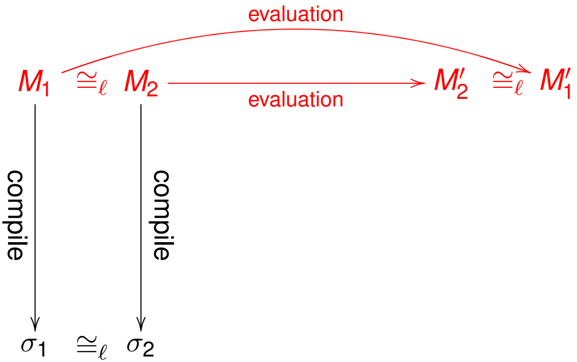

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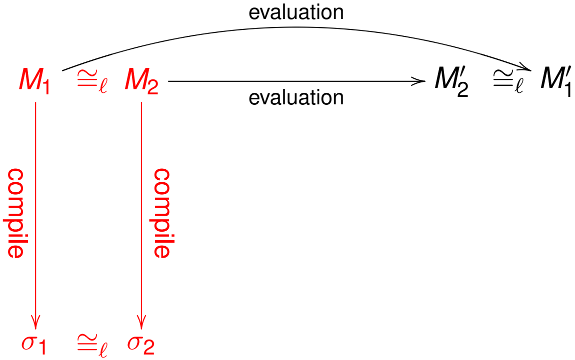

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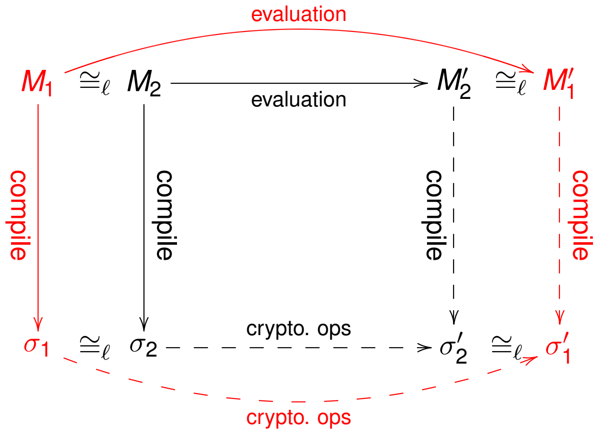

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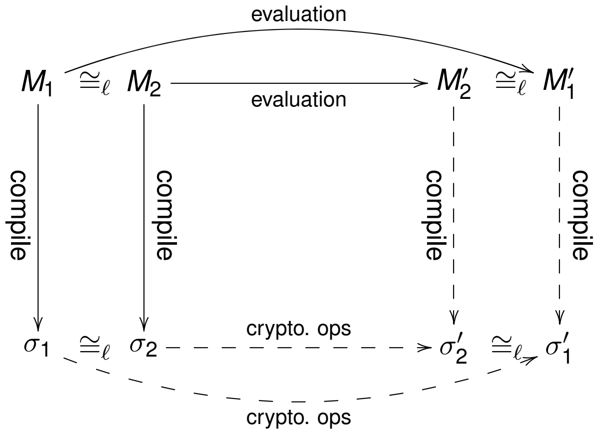

Take home messages

SImp explores a new space in information flow languages with

- declarative policies implemented by a cryptographic mechanism,
- a strong noninterference property,
- and a rich, structural label language.

Appendix

- Typing Rules
- Future Work
- Package Uniqueness
- DLM Comparison
- Expression Noninterference Statement
- Command Noninterference Statement
- Adequacy Statements
- Feasibility Statement
- Cryptographic Operations

Pack provides a limited declassify, unpack an endorse.

Definition Fragment

$$\frac{\Theta; \Gamma \vdash e : \tau\{\ell_e\} \qquad I(\ell_e) = I(\ell)}{\Theta; \Gamma \vdash \texttt{pack}\ e\ \texttt{at}\ \ell_e : (\texttt{pkg} + \texttt{error})\{\ell\}}$$

$$\frac{\Theta; \Gamma \vdash e : \texttt{pkg}\{\ell_e\} \qquad C(\ell_e) = C(\ell)}{\Theta; \Gamma \vdash \texttt{unpack}\ e\ \texttt{as}\ \tau\{\ell\} : (\tau + \texttt{error})\{\ell\}}$$

These rules are safe because of the dynamic checks.

This is just the beginning.

Further research questions:

- Can homomorphic encryption be used for computation within packages?
- How can we compile alternative label models?
  - “share semantics”
  - uniqueness labels
- How does package upgrading and downgrading interact with cryptography?

A explicit non-goal: package uniqueness.

- Replay attacks (vs. legitimate uses of persistence) are best detected at higher levels of abstraction.
- Uniqueness checks appear to require interactive protocols.
- Resolving these challenges would be interesting future work.

Our DLM is a variant of Myers and Liskov's original.

Example

$$\ell = \{\mathsf{Alice}: \ \mathsf{Charlie}\,!\,\emptyset;\ \mathsf{Bob}: \ \mathsf{Dave}\,!\,\emptyset\}$$

- Here:

$$\vdash \{\mathsf{Charlie}, \mathsf{Dave}\}\ \textit{reads}\ \ell$$

- Myers and Liskov:

$$\not\vdash \{\mathsf{Dave}, \mathsf{Charlie}\}\ \textit{reads}\ \ell$$

We don't consider an explicit *acts-for-hierarchy*.

- It should work technically but is orthogonal.
- Intuitively, principal sets "act for" component principals.
- Key difference:
  - Myers and Liskov: Calculate readers, then close under acts-for.
  - Here: Close under acts-for, then calculate readers.

Formal statement of expression noninterference.

Theorem (Expression noninterference)

If

- $\Theta \vdash M_1 \ OK$, $\Theta \vdash M_2 \ OK$ *and* $\Theta \vdash M_1 \cong_\ell M_2$
- $\Theta; \cdot \vdash e_1 : \tau\{\ell_e\}$ *and* $e_1 \cong_\ell e_2$ *where* $\ell_e \leq \ell$
- $\overline{p}; M_1 \vdash e_1 \longrightarrow^* v_1$ *and* $\overline{p}; M_2 \vdash e_2 \longrightarrow^* v_2$

then $v_1 \cong_\ell v_2$.

Command evaluation respects noninterference.

Theorem (Command Noninterference)

If

- $\Theta \vdash M_1$ *OK*, $\Theta \vdash M_2$ *OK and* $\Theta \vdash M_1 \cong_\ell M_2$
- $pc; \Theta; \cdot \vdash c_1$ *and* $c_1 \cong_\ell c_2$
- $\overline{p} \vdash \langle M_1, c_1 \rangle \longrightarrow^* \langle M_1', \texttt{skip} \rangle$ *and*
  $\overline{p} \vdash \langle M_2, c_2 \rangle \longrightarrow^* \langle M_2', \texttt{skip} \rangle$

then $\Theta \vdash M_1' \cong_\ell M_2'$.

Adequacy theorems: compilation is secret preserving.

Lemma (Adequacy of Value Translation)

If $v_1 \cong_\ell v_2$ *and* $\overline{\kappa}$ *is fresh then* $\mathrm{V}[\![v_1]\!]_{\overline{\kappa}} \cong_\ell \mathrm{V}[\![v_2]\!]_{\overline{\kappa}}$.

Corollary (Adequacy of Memory Translation)

If $\Theta \vdash M_1 \cong_\ell M_2$ *and* $\overline{\kappa}$ *is fresh then* $\mathrm{M}[\![M_1]\!]^{\Theta}_{\overline{\kappa}} \cong_\ell \mathrm{M}[\![M_2]\!]^{\Theta}_{\overline{\kappa}}$.

Realizable operations simulate SImp evaluation.

Theorem (Feasibility)

If

- $\Theta \vdash M\ OK$
- $pc; \Theta; \Gamma \vdash c$
- $\overline{p}$ *reads pc and* $\overline{p}$ *writes pc,*
- $\overline{p} \vdash \langle M, c \rangle \longrightarrow \langle M', c' \rangle$

then

$\exists \overline{\kappa}_3, \overline{\kappa}_4.\ \vdash \mathrm{M}[\![M]\!]^{\Theta}_{\overline{\kappa}_1} \cup \mathit{state}(\overline{\kappa}_2, \overline{p}, c) \longrightarrow^* \mathrm{M}[\![M']\!]^{\Theta}_{\overline{\kappa}_3} \cup \mathit{state}(\overline{\kappa}_4, \overline{p}, c)$.

Only some operations are cryptographically realizable.

Definition

$$\frac{}{\vdash \sigma \rightarrow \sigma, \mathit{knows}\ (K_\kappa^+, K_\kappa^-)}\ \textsc{cs-Fresh} \quad \kappa \text{ fresh}$$

$$\textsc{cs-Derive}\ \frac{\sigma \vdash_d m}{\vdash \sigma \rightarrow \sigma, \mathit{knows}\ m} \qquad \textsc{cs-Forget}\ \frac{\sigma' \subseteq \sigma}{\vdash \sigma \rightarrow \sigma'}$$

$$\textsc{cs-Compute}\ \frac{\sigma \vdash_d \texttt{"i}_1\texttt{"} \qquad \sigma \vdash_d \texttt{"i}_2\texttt{"}}{\vdash \sigma \rightarrow \sigma, \texttt{"i}_3\texttt{"}} \text{ where } i_3 = i_1 + i_2$$