# Subtyping and the Liskov Substitution Principle

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# Syntactic and Semantic Properties

## Syntactic Properties

Program properties program defined by “simple” formal rules, and automatically checked.

- Are all parentheses matched?
- Does the program contain type errors?
- Are there are undefined variables?

## Semantic Properties

Program properties defined with arbitrary reasoning.

- Do these methods calculate the same function?
- What is the asymptotic complexity of this method?
- Does this loop satisfy an invariant?

# Syntactic and Semantic Properties

## Syntactic Properties

Program properties program defined by “simple” formal rules, and automatically checked.

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- Does the program contain type errors?
- Are there are undefined variables?
- Is class A derived from class B?

## Semantic Properties

Program properties defined with arbitrary reasoning.

- Do these methods calculate the same function?
- What is the asymptotic complexity of this method?
- Does this loop satisfy an invariant?
- Is it safe to treat an instance of A like a B?

# Subtyping

Say A is a *subtype* of B when an instance of A can be used in place of an instance B.

Example

```
class Math {
  virtual int Add(int x,int y){ return x + y; } }

// More math is a subtype of Math
class MoreMath : Math {
  virtual int Divide(int x, int y){ return (x/y); }

// What is EvenMoreMath a subtype of?
class EvenMoreMath : MoreMath {
  override int Add(int x,int y){ return 399; } }
```

# Two views of subtyping

- Syntactic view: A `is_a` B when A extends or implements B.
  - Ensures type safety: Anyone expecting a B will find A has the appropriate members.
  - Relatively easy to check.
  - But does not ensure programs will work correctly.

- Semantic view: A `<:` B when instances of A exhibit behavior equivalent to instances of B in places where Bs are expected.
  - A `is_a` B necessary condition for A `<:` B
  - Rules out many errors possible with only syntactic subtyping.
  - But impossible to enforce automatically.

# Back to the even more EvenMoreMath example

- EvenMoreMath is a syntactic subtype of Math and MoreMath.
- EvenMoreMath is a semantic subtype of Math.
- Is EvenMoreMath a semantic subtype of MoreMath?.
  - Depends on the specification of MoreMath and EvenMoreMath…

# Method specification subtyping

Before working out class subtyping, need to figure out when method specifications can be considered subtypes.

Use precise specifications to help determine subtyping relation.

Key Idea: m `<:` n when m

- accepts more inputs than n
- produces fewer potential outputs than n
- otherwise obeys the specification of n.

N.B. We will be using some Java 1.5 features that have not made it into C#.

# Subtyping and method results

When is m $<:$ n?

```
// returns: An assumption about the returned S
S m(X in)

// returns: An assumption about the returned T
T n(X in)
```

If some program expects an n, but is given an m, things will be ok when both:

# Subtyping and method results

When is m $<:$ n?

```
// returns: An assumption about the returned S
S m(X in)

// returns: An assumption about the returned T
T n(X in)
```

If some program expects an n, but is given an m, things will be ok when both:

- S $<:$ T, and
- assumptions about the returned S are *stronger* (i.e. more specific, or more restrictive) than assumptions about the returned T.

# Example

Assume $W <: U$

```
// a() returns some W
W a(X in)

// b() returns some W != null
W b(X in)

// c() returns some U
U c(X in)

// d() returns some U != null
U d(X in)
```

What subtypes are here?

# Example

Assume W $<:$ U

```
// a() returns some W
W a(X in)

// b() returns some W != null
W b(X in)

// c() returns some U
U c(X in)

// d() returns some U != null
U d(X in)
```

What subtypes are here? b $<:$ a, a $<:$ c, b $<:$ d, d $<:$ c...
but a and d are not subtypes of each other.

# Subtyping and method inputs

When is m $<:$ n?

```
// requires: An assumption about the X input
T m(X in)

// requires: An assumption about the Y input
T n(Y in)
```

If some program expects an n, but is given an m, things will be ok when:

# Subtyping and method inputs

When is m $<:$ n?

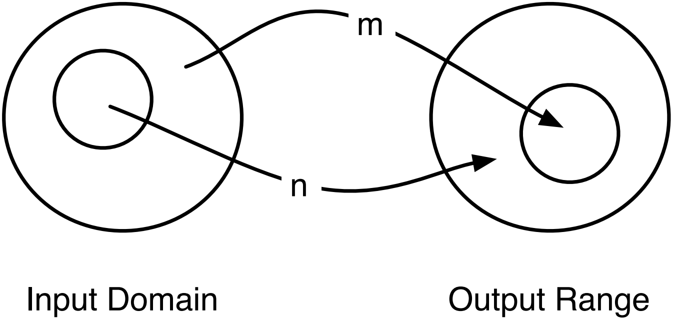
```
// requires: An assumption about the X input
T m(X in)

// requires: An assumption about the Y input
T n(Y in)
```

If some program expects an n, but is given an m, things will be ok when:

- Y $<:$ X, and
- assumptions about the input Y are *stronger* than assumptions about the returned X.
- *Opposite of returns*
    - Returns—“covariant”
    - Requires—“contravariant”

# Method Specification Subtyping Pictorially

m <: n



# Checks vs. Requires

- //requires—anything can occur given bad input
- //checks—an exception must be thrown on bad inputs
- checks is more specific

```
//checks x > 0
void m(int x)

//requires x > 0
void n(int x)
```

m <: n

# Effects must be “invariant”.

```
//A counter is part of some abstract state.

// Effects: increment the counter
void m()

// Effects: decrements the counter
void n()
```

Because m and n have different effects, we cannot replace one by the other:

$$m \not<: n \text{ and } n \not<: m$$

# Class/Interface Semantic Subtyping

$A <: B$ for classes/interfaces A and B

- For all accessible members, m,

  $$A.m <: B.m$$

- Each abstract state of A is part of an abstract state of B.
  - Otherwise code trying to use an A in place of a B will be confused.

# Example

```
// State: amount is EMPTY or FULL
class Glass{ ... }

// State: amount is EMPTY or FULL;
//        kind is ORANGE or APPLE
class JuiceGlass{ ... }

// State: amount is EMPTY, HALF or FULL
class PreciseGlass{ ... }

// State: amount is EMPTY, HALF or FULL
//        kind is ORANGE or APPLE
class PreciseJuiceGlass{ ... }
```

Subtypes:

# Example

```
// State: amount is EMPTY or FULL
class Glass{ ... }

// State: amount is EMPTY or FULL;
//        kind is ORANGE or APPLE
class JuiceGlass{ ... }

// State: amount is EMPTY, HALF or FULL
class PreciseGlass{ ... }

// State: amount is EMPTY, HALF or FULL
//        kind is ORANGE or APPLE
class PreciseJuiceGlass{ ... }
```

Subtypes: JuiceGlass $<:$ Glass, Glass $<:$ PreciseGlass,
JuiceGlass $<:$ PreciseJuiceGlass,
PreciseJuiceGlass $<:$ PreciseGlass

(Adding methods may reduce number of subtypes)

# The Liskov Substitution Principle

A `is_a` B only makes sense when A `<:` B

Alternatively: Let q(x) be a property provable about objects x of type T. Then q(y) should be true for objects y of type S where S is a subtype of T.

# Thoughts on the Liskov Substitution Principle

- Hard to design an maintain classes that respect the `<:` relation.
- Classes hierarchies that don’t respect `<:` are likely to
  - contain subtle bugs
  - requires lots of type tests using is or as
  - both
- In practice we see lots of shallow class hierarchies—is this due to the difficulty of building Liskov substitutable classes?